BASIC CONCEPTS INVOLVED IN FACTORING TRINOMIALS

- Need some foundational material first?
  For a basic discussion of factors: Recognizing Products and Sums; Identifying Factors and Terms
  For practice factoring out a greatest common factor: Factoring out the Greatest Common Factor
  Also, you need to be ***completely comfortable*** with Addition of Signed Numbers.

(more mathematical cats)

Here, you will practice the basic concepts involved in factoring trinomials of the form $x^2 + bx + c$.
These trinomials have an x^2 term with a coefficient of 1, an x term, and a constant term.

Recall that ***factoring*** is the process of taking a sum (things added) and rewriting it as a product (things multiplied).

Observe that for all real numbers f, g and x:

$$(x+f)(x+g) = \overbrace{x^2}^{\text{First}} + \overbrace{gx}^{\text{Outer}} + \overbrace{fx}^{\text{Inner}} + \overbrace{fg}^{\text{Last}} = x^2 + (f+g)x + fg$$

Now, think of going ‘backwards’:
from $x^2 + (f+g)x + fg$
back to the factored form $(x+f)(x+g)$.

We'd need two numbers that ***add together*** to give the coefficient of the x term,
and that ***multiply together*** to give the constant term.

This gives the following result, which is the primary tool used in factoring trinomials:

KEY TOOL FOR FACTORING TRINOMIALS
(where the coefficient of the squared term is 1*)*

To factor a trinomial of the form $x^2 + bx + c$,
start by finding two numbers, f and g, that

- ***add together*** to give b (the coefficient of the x term); and
- ***multiply together*** to give c (the constant term).

Then:

$$x^2 + bx + c \;=\; x^2 + (\overbrace{f+g}^{=b})x + \overbrace{fg}^{=c} \;=\; (x+f)(x+g)$$

For example, to factor $x^2 + 5x + 6$,
we must find two numbers that add to 5 and multiply to 6.
The numbers 2 and 3 work, since $2 + 3 = 5$ and $2 \cdot 3 = 6$.

Thus:

$$x^2 + 5x + 6 \;=\; x^2 + (2+3)x + (2 \cdot 3) \;=\; (x+2)(x+3)$$

(FOIL it out to check!)

When everything in sight is positive and coefficients are small,
then it may be easy to come up with the ‘numbers that work’.
For example, it may not be too hard for you to find numbers that add to 5 and multiply to 6.

However, bring some negative numbers into the picture and make coefficients bigger,
and things can get considerably trickier.

Fortunately, there are some key ideas that will help you find the ‘numbers that work’ (if they exist),
and the purpose of this web exercise is to give you practice with these ideas.

KEY IDEAS FOR FINDING THE ‘NUMBERS THAT WORK’:

- If two numbers multiply to give a POSITIVE number,
  then they must both be positive or they must both be negative.
  That is, ***two numbers that multiply to a positive number*** must have ***the same sign***.

- If two numbers multiply to give a NEGATIVE number,
  then one must be positive, and the other must be negative.
  That is, ***two numbers that multiply to a negative number*** must have ***different signs***.

- When you ***add two numbers that have the same sign***,
  then ***in your head you do an addition problem***.

  For example, to mentally compute the sum $(-2) + (-5)$,
  in your head you would compute $2 + 5$, and then assign a negative sign to your answer.

  Think of the number line interpretation of this fact.
  Start at zero.
  You must walk in only one direction (both numbers have the same sign).
  Each time you walk, you're getting farther from zero.
  Your final distance from zero is the sum of the individual distances you walk.

- When you ***add two numbers that have different signs***,
  then ***in your head you do a subtraction problem***.

  For example, to mentally compute $-5+2$,
  you would ***think*** $5-2$, and then make the answer negative.

  Think of the number line interpretation of this fact.
  Start at zero.
  You must walk in both directions (the numbers have different signs).
  So, you'll be doing some back-tracking (some overlapping).
  Your final distance from zero is the difference of the individual distances you walk.

- If two numbers have the same sign and their sum is positive,
  then both numbers must be positive.

  For example, if two numbers have the same sign and add to 10,
  then they must both be positive.

  Think of the number line interpretation of this fact.
  Start at zero.
  You need to walk in only one direction (both numbers have the same sign).
  You need to end up to the right of zero (the sum is positive).
  So, you must walk to the right both times (both numbers must be positive).

- If two numbers have the same sign and their sum is negative,
  then both numbers must be negative.

  For example, if two numbers have the same sign and add to -10,
  then they must both be negative.

  Think of the number line interpretation of this fact.
  Start at zero.
  You need to walk in only one direction (both numbers have the same sign).
  You need to end up to the left of zero (the sum is negative).
  So, you must walk to the left both times (both numbers must be negative).

- If two numbers have different signs and their sum is positive,
  then the bigger number must be positive.
  (Remember that ‘bigger’ means ***farther away from zero***.)

  For example, if two numbers have different signs and add to 10,
  then the bigger number must be positive.
  (Like $12 + (-2) = 10$: the numbers being added are 12 and -2;
  12 is bigger because it is farther from zero.)

  Think of the number line interpretation of this fact.
  Start at zero.
  You must walk in both directions (the numbers have different signs).
  You need to end up to the right of zero (the sum is positive).
  So, you must walk farther to the right (the bigger number must be positive).

- If two numbers have different signs and their sum is negative,
  then the bigger number must be negative.
  (Remember that ‘bigger’ means ***farther away from zero***.)

  For example, if two numbers have different signs and add to -10,
  then the bigger number must be negative.
  (Like $-12 + 2 = -10$: the numbers being added are -12 and 2;
  -12 is bigger because it is farther from zero.)

  Think of the number line interpretation of this fact.
  Start at zero.
  You must walk in both directions (the numbers have different signs).
  You need to end up to the left of zero (the sum is negative).
  So, you must walk farther to the left (the bigger number must be negative).

EXAMPLES:

Question: Suppose two numbers multiply to 36.
Then, the numbers have (choose one):

THE SAME SIGN	DIFFERENT SIGNS

Answer: THE SAME SIGN

Question: Suppose two numbers multiply to -36.
Then, the numbers have (choose one):

THE SAME SIGN	DIFFERENT SIGNS

Answer: DIFFERENT SIGNS

Question: Suppose two numbers have the same sign, and they add to 10.
Then, the numbers are (choose one):

BOTH POSITIVE	BOTH NEGATIVE

Answer: BOTH POSITIVE

Question: Suppose two numbers have the same sign, and they add to -10.
Then, the numbers are (choose one):

BOTH POSITIVE	BOTH NEGATIVE

Answer: BOTH NEGATIVE

Question: When you add two numbers that have the same sign,
then in your head you do a/an (choose one):

ADDITION PROBLEM	SUBTRACTION PROBLEM

Answer: ADDITION PROBLEM

Question: When you add two numbers that have different signs,
then in your head you do a/an (choose one):

ADDITION PROBLEM	SUBTRACTION PROBLEM

Answer: SUBTRACTION PROBLEM

Question: Suppose two numbers have different signs, and they add to 10.
Then, the bigger number is (choose one):

POSITIVE	NEGATIVE

Answer: POSITIVE

Question: Suppose two numbers have different signs, and they add to -10.
Then, the bigger number is (choose one):

POSITIVE	NEGATIVE

Answer: NEGATIVE